Indian J. Phys. **78A** (3), 353-357 (2004)

AN INTERNATIONAL JOURNAL **IJP A**

Pion fluctuation study in π^-–AgBr interactions at 350 GeV/c

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Received 29 January 2003, accepted 4 July 2003

Abstract : Fluctuations in pseudorapidity distribution of secondary particles in π^-–AgBr interactions at 350 GeV/c are analysed using F_q moments, G_q moments and T_q moments. In order to investigate the presence of non-thermal phase transition, the required parameters have been determined from these three moments and are compared among themselves with orders

Keywords : Factorial moment, G_q moment, Takagi moment, non-thermal phase transition

PACS Nos. : 13.85.Hd, 25.80.-e.

1. Introduction

In rapidity densities of charged secondaries, the study of non-statistical fluctuations have recently received much attention due to its potential to provide a better understanding of multiparticle production mechanism. The concept of intermittency was first introduced in the field of multiparticle production by Bialas and Peschanski [1-3] from the theory of turbulence where it is used to measure the effects of bursts in a turbulent system. Specifically, they suggested that if intermittency exists, the normalized factorial moments of multiplicity distribution should exhibit power-law dependence on the width $\delta\eta$ of the rapidity window, as $\delta\eta \to 0$. This point of view of Bialas and Peschanski received a tremendous boost of support when intermittent behaviour were observed in e^+e^- annihilation [4], hadron-hadron [5], hadron-nucleus [6] and nucleus-nucleus [7] interactions. In each case, a power-law dependence of the scaled factorial moments of the multiplicity distribution on the rapidity bin width was observed and this type of multiplicity fluctuation was called intermittency.

The power-law behaviour of factorial moments reveals self-similarity and in general, indicates the existence of fractal properties [8] in multiparticle production process. For investigating the fractal structure in multiparticle data, various methods have been suggested. Hwa [9] proposed the G_q moment approach which has enriched they study of multifractal analysis in multiparticle production. However, a crucial point to be stressed here is that the power-law behaviour of G_q is to be analyzed event by event, and not for the event average which suppresses the importance of fluctuation in low multiplicity events. Hwa and Pan [10] modified the old forms of the G_q moments by introducing a step function which can act as a filter for the low multiplicity events. Although for investigation of fractal structure, anothe moment T_q has also been suggested by Takagi [11]. In this particular approach, the experimental data present the expected linear behaviour in a log-log plot within the required mathematical limit, the number of points $\to \infty$. T_q moment approach is different from both F_q moment and G_q moment approach. A crucial difference lies in the fact that the total number of points (particles) can be made arbitrarily large.

The motivation of this paper is to investigate the signal of non-thermal phase-transition in π^-–AgBr interactions at 350 GeV/c. In particular, approaches based on the concept of (Multi) fractals seem to be most interesting as they may be related to phase transitions [12], self-similarity cascade *etc.* Using these three methods, we can try to gain some knowledge about possible phase transitions. One should not necessarily imply the transition from a hadron phase to quark-gluon plasma but can also speculate about other possible transitions due to symmetry breaking in the system.

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2. Experimental data

We study the hadron nucleus interaction data of π^--AgBr at 350 GeV/c incident energy. The stacks of G5 nuclear emulsion plates were exposed horizontally to a π^- beam at CERN with 350 GeV/c incident energy. The nuclear emulsion covers 4π geometry and provides very good accuracy (even less than 0.1 m rad) in angle measurements of produced particles with respect to the projectile beam axis due to high spatial resolution and thus, is suitable as a detector for the study of fluctuations in fine resolution intervals of the pseudo-rapidity space. The emulsion plates were area scanned with Leitz Metalloplan Microscope fitted with a semi automatic scanning device, having a resolution along with the X and Y axes of 1 μm while that along with the Z axis is 0.5 μm. A sample of 569 events of π^--AgBr at 350 GeV/c was chosen by the usual emulsion methodology. The details of the experimental data set has been described in our earlier paper [13]. The average number of shower tracks for the data sample used, is 11.74 ± 0.12.

The emission angle θ of each produced secondary was measured by recording the space coordinates (x, y, z) of a point on each shower track, the point of interaction and a point on the incident track. Pseudo-rapidity η of all shower particles were determined from measured space angle θ with reference to the beam by the relation

$$\eta = -\ln \tan(\theta/2). \tag{1}$$

3. Method of analysis

The methods based on F_q moment, G_q moment and T_q moment are explained in this section.

(A) F_q moments method :

For the analysis of fluctuations in pseudorapidity distribution, the pseudorapidity interval $\Delta\eta$ is divided into M bins of size $\delta\eta = \Delta\eta / M$. The q-th order scaled factorial moment is defined by [1].

$$\langle F_q \rangle = \frac{1}{N_{ev}} \sum_N M^{q-1} \sum_{m=1}^{M} \frac{n_m(n_m-1)\ldots(n_m-q+1)}{\langle N \rangle^q}, \tag{2}$$

where N_{ev} is the number of events in the sample, n_m is the number of showers in bin m and <N> is the mean multiplicity of showers in the $\Delta\eta$ interval. The dependence of the scaled factorial moments on bin width $\delta\eta$ is a measure of the size of the pseudorapidity fluctuations. An intermittent pattern of the fluctuations leads to a power-law behaviour of the moments [1] given by

$$\langle F_q \rangle \propto \left[\frac{\Delta\eta}{\delta\eta}\right]^{\alpha_q}, \tag{3}$$

where the slope value α_q characterizes the strength of intermittency.

Fialkowski *et al* [14] suggested that scaled factorial moments of eq. (2) should be corrected to take into account of the non-uniform shape of the single-particle pseudorapidity distribution. The corrected scaled factorial moments are given by

$$\langle F_q \rangle_{corr} = \langle F_q \rangle / R_q, \tag{4}$$

where R_q is the q-th order correction factor given by

$$R_q = \frac{1}{M} \sum_{m=1}^{M} M^q \frac{\langle n_m \rangle^q}{\langle N \rangle^q} \tag{5}$$

We have evaluated the corrected scaled factorial moments $\langle F_q \rangle_{corr}$.

(B) G_q moments method :

For investigating multiparticle fluctuations, the method of fractal moment G_q, determined to be

$$\langle G_q \rangle = \sum_{m=1}^{M} (n_m / N)^q \, \theta(n_m) \tag{6}$$

has been recently suggested by Hwa [9]. Here, an order q is an arbitrary real number, n_m is the produced partricle multiplicity in the m-th bin, $\delta\eta (= \Delta\eta / M)$ is the pseudorapidity interval $\Delta\eta$ divided into M bins and N is the total multiplicity of the considered event given by

$N = \sum_{m=1}^{M} n_m$ and $\theta(\eta_m)$ is the (Heaviside) step function.

The modified form of multifractal multiplicity moments of order q proposed by Hwa and Pan [10] is defined for a single event by

$$\langle G_q \rangle = \sum^{M} (n_m / N)^q \theta(n_m - q) \tag{7}$$

instead of eq. (6). At very large N, the new definition coincides with eq. (6). Since in reality, N is limited, the presence of different θ-functions gives different results.

To get the statistical contribution to $\langle G_q \rangle$, N-particles of each event are distributed randomly throughout the considered η-interval and the value of $\langle G_q^{st} \rangle$ moment is obtained with the redistributed particles.

For self similar processes, $\langle G_q \rangle$ are the power-law functions of the witdh $\delta\eta$

$$\langle G_q \rangle \propto M^{-\tau_q} \propto (\delta\eta)^{\tau_q}. \tag{8}$$

After the average overall events of quantities of $\langle \ln G_q$ one gets

$$\tau_q = -\frac{\Delta\langle \ln G_q \rangle}{\Delta \ln M} \tag{9}$$

in the range of values of M for which the self-similar behaviour (8) of the $\langle G_q \rangle$ moment exists.

The slopes τ_q characterize multifractal properties of the pseudorapidity distributions and it was revealed that the slopes of dynamical τ_q^{dyn} and statistical τ_q^{st} contributions are connected with each other by the relation

$$\tau_q^{dyn} = \tau_q - \tau_q^{st} + q - 1 , \tag{10}$$

where the τ_q slopes include both statistical and dynamical ones.

(C) T_q moments method :

Takagi [11] proposed a method to study the process of multiparticle production at some incident energy and the distribution in the rapidity space. A single event contains n hadrons distributed in the interval $\eta_{\min} < \Delta\eta < \eta_{\max}$. The multiplicity n changes from event to event according to the distribution $P_n(\eta)$ where $\Delta\eta = \eta_{\max} - \eta_{\min}$. Divide the full rapidity interval of length $\Delta\eta$ into M bins of equal size $\delta\eta = \Delta\eta / M$. The multiplicity distribution for a single bin is denoted as $P_n(\delta\eta)$ for $n = 0, 1, 2$........., where Takagi [11] assumed that the inclusive rapidity distribution $dn/d\eta$ is constant and $P_n(\delta\eta)$ is independent of the location of the bin. Let K be total number of hadron produced and n_{ij} the multiplicity of hadron in the j-th bin of the i-th event. The theory of multifractals motivated Takagi to consider the normalized density P_{ij} defined by

$$P_{ij} = n_{ij} / K . \tag{11}$$

Hadron produced in Ω independent events are distributed in ΩM bins of size $\delta\eta$ and the method of fractal moment $T_q(\delta\eta)$ may be determined as

$$T_q(\delta\eta) = \ln \sum_{i=1}^{\Omega} \sum_{j=1}^{M} P_{ij}^q \quad \text{for } q > 0. \tag{12}$$

Evaluating the double sum of P_{ij}^q for sufficiently large Ω and using $\Omega M = K / < n >$, Takagi [11] indicated a linear relation like

$$\ln\langle n^q \rangle = A_q + \left(B_q + 1\right) \ln < n > , \tag{13}$$

where A_q and B_q are constants independent of $\delta\eta$.

4. Results and discussion

To get an insight into the production mechanism at work in hadron-nucleus interactions, we have studied various parameters which serve as signatures for describing the origin of the observed non-thermal behaviour. These parameters are calculated from the slope values of F_q moments, G_q moments and T_q moments. The errors shown with the data for F_q, G_q and T_q moments are statistical in nature. One should note that it is not easy to calculate these errors because in most cases, the points are correlated [15]. However, the inclusion of their contribution does not change the main result appreciably as shown in Ref. [16].

A linear rise of $\ln\langle F_q \rangle$ with $\ln M$ is observed which exhibits the presence of intermitted behaviour shown in the Figure 1.

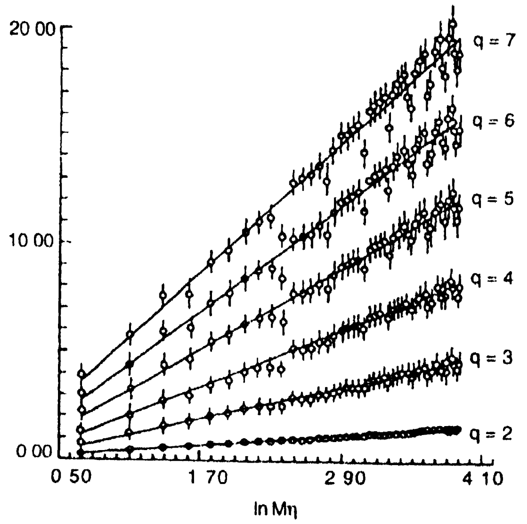

Figure 1. $\ln<F_q>_{corr}$ as a function of lnM for q = 2 to 7. Lines represent the linear fits to the data.

The slope value α_q, a characteristic parameter of intermittency is computed by a linear fit of our data in pseudorapidity space to the relation

$$\ln\langle F_q \rangle_{corr} = \alpha_q \ln M + C_q .$$

Table 1 shows the slope values of α_q for corrected scaled factorial moments with order q.

Table 1. Using F_q moments, the fitted slope parameters α_q with order q found in π-AgBr interactions at 350 GeV/c

Order q	2	3	4	5	6	7
α_q	0.637	1.382	2.141	2.892	3.655	4.408
	± 0.004	± 0.009	± 0.017	± 0.039	± 0.056	± 0.090

We plot $\ln\langle n^q \rangle$ as a function of ln<n> shown in Figure 2. Our experimental data exhibit the linear behaviour in the log-log

Table 2. Fitted slope parameter B_q of T_q moments analysis with order q found in π^--AgBr interactions at 350 GeV/c.

Order q	2	3	4	5
B_q	0.613	1.141	1.658	2.186
	± 0.02	± 0.02	± 0.03	± 0.02

plot and now there is no problem to calculate the slope values of the linear fit. Table 2 presents the slope value B_q of T_q moments.

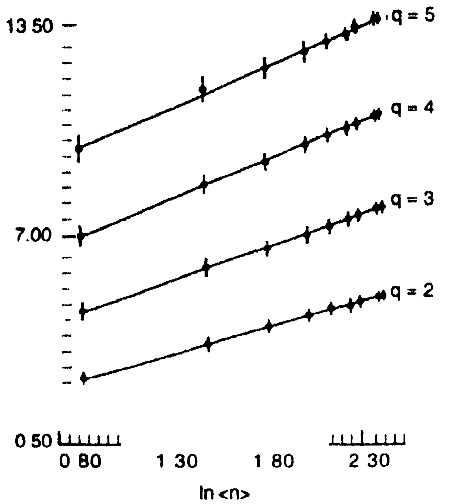

Figure 2. ln<n^q> as a function of ln<n> for q = 2 to 5. Lines represent the linear fit to the data

Figure 3 depicts the variation of $\ln\langle G_q\rangle$ with lnM. A linear dependence of $\ln\langle G_q\rangle$ on ln$M$ is observed indicating self-similarity in particle production process. The best fit curves of our data are drawn as smooth straight lines in Figure 3. The fitted slope values τ_q, τ_q^{st} and τ_q^{dyn} are listed in the Table 3.

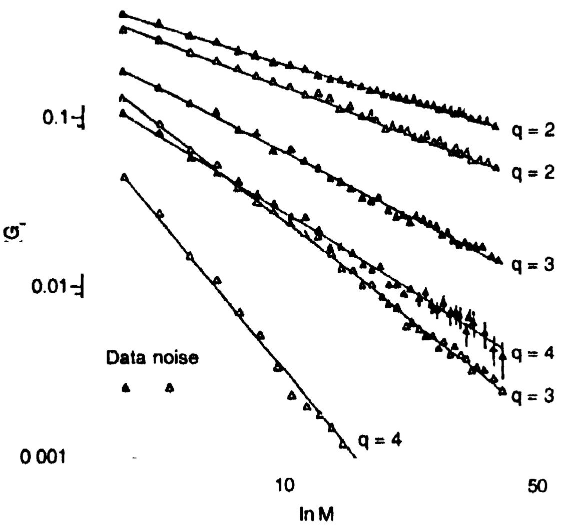

Figure 3. ln<G_q> as a function of lnM for q = 2 to 4. Lines represent the linear fit to the data

Table 3. Fitted slope parameters $\left(\tau_q, \tau_q^{st}, \tau_q^{dyn}\right)$ of G_q moment analysis with order q in π^--AgBr interactions at 350 GeV/c.

Order q	2	3	4
τ_q	0.66 ± 0.02	1.15 ± 0.04	1.38 ± 0.06
τ_q^{st}	0.84 ± 0.02	1.74 ± 0.03	2.76 ± 0.08
τ_q^{dyn}	0.82 ± 0.03	1.41 ± 0.05	1.62 ± 0.10

The dynamical part τ_q^{dyn} of τ_q has been extracted according to the formula (10) for each order of moments and is related to the generalized fractal dimension D_q^G by

$$D_q^G = \tau_q^{dyn}/(q-1). \tag{14}$$

Similarly, there is no difficulty in determining the generalized fractal dimension D_q^T with the help of the slopes of the linear eq. (13) by

$$D_q^T = B_q/(q-1). \tag{15}$$

Multifractality exists if the generalized fractal dimension increases with the decrease of the order. Figure 4 shows the q dependence of D_q for each moment satisfies the condition of multifractality. It is also observed that for G_q moments the plot in D_q^G with q is steepy than that of other two moments

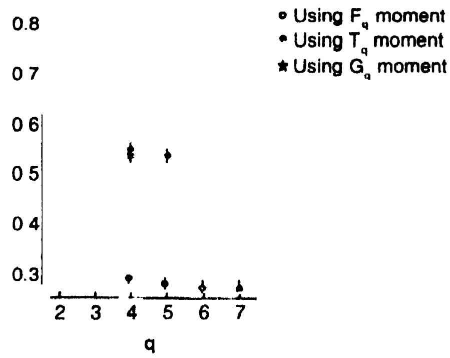

Figure 4. Generalised fractal dimension D_q as a function of order q in different moments.

The relation between generalized dimension and anomalous fractal dimension is given by definition

$$d_q = 1 - D_q, \tag{16}$$

where $d_q = \alpha_q/(q-1)$. (17)

From eqs. (14)-(17), one gets

$$\alpha_q^G = q - 1 - \tau_q^{dyn} \tag{18}$$

and $\alpha_q^T = q - 1 - B_q$. (19)

The relationship among α_q, α_q^G and α_q^T are different because the approaches of different moments are different Figure 5 exhibits that the values of α_q, α_q^G and α_q^T are not exact but all of them increase with order q.

Peschanski has suggested that the self-similar cascade can occur in different phases [17], the normal phase populated by many relatively small fluctuations and the spin-glass phase consisting of a few very large fluctuations. The condition for the coexistence of these two phases of the cascade is the existence of a minimum of the parameter λ_q at a certain value of the order $q = q_c$. The value of λ_q is defined by

$$\lambda_q = (\alpha_q + 1)/q \,. \tag{20}$$

The regions $q < q_c$ and $q > q_c$ correspond to the normal and the spin-glass phase respectively.

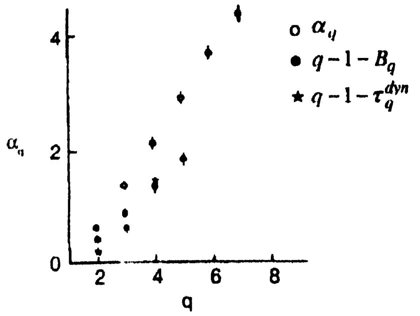

Figure 5. Comparison of the exponents of F_q moments, G_q moments (pollution free) and T_q moments.

We can easily calculate the parameter λ_q with the help of the eq (20) for these three types of moments. The variation of λ_q with order q is shown in Figure 6. Our data with F_q moments and T_q moments show flattening around $q = 4$, but no clear minimum is seen. On the other hand, a minimum of λ_q for G_q moments for a certain value $q = 3$ is observed. The observation of minimum may be an indication of the presence of non-thermal phase transition.

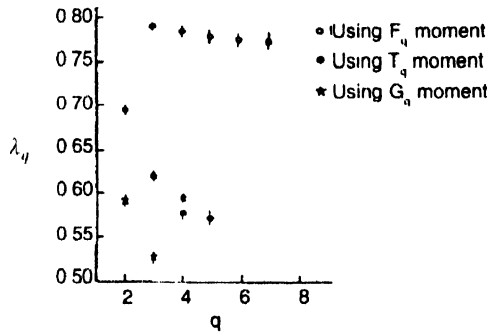

Figure 6. Parameter λ_q as a function of order q in different moments

5. Conclusion

We end with the concluding remark that in hadron-nucleus interactions at 350 GeV/c, the observed pattern of λ_q against q gives no definite indication for the existence of two phases.

Acknowledgment

The authors would like to thank Prof. W Lock of CERN for providing us with the exposed and developed emulsion plates used for this work. Authors also gratefully acknowledge the financial help from the University Grants Commission (India) under the COSIST programme.

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